Introduction

From the stimulus sports I chose to centre my internal assessment on basketball. Basketball is of great interest to me as it is the sport I most enjoy playing. Being on the school basketball team for four years has enabled me to see that there is a connection between the field of math and the sport. However, what is that connection exactly and can we use math to aid us in improving our plays in basketball?

I decided to look at basketball from an algebraic perspective where the basketball forms a curve when thrown. The variables used will be the different heights of the factors involved in the scenario as well as the distances between these factors. Through these variables I will find an equation for the parabola or curve and I can use it to determine where I should stand on the court to make the play. Different scenarios will be analyzed mathematically, one where I am the shooter and one where I am the player blocking the ball.
Scenario A

In this scenario I am the shooter and I have to throw the ball past my opponent who is attempting to block the ball. To find the equation I must first find all the variables.

- The distance between me and the basket is 10 meters
- The distance between me and my opponent is 3 meters
- The distance between my opponent and the basket is therefore 7 meters
- My height is 1.8 meters
- My opponent’s height while jumping with his/her arms up in blocking position is 3 meters
- The height of the basket is 2.5 meters
- The height of the basket above my head is 0.7 meters because: 2.5 (the height of the basket) – 1.8 (my height) = 0.7 meters
- The height above my head till my opponent’s fingers while he’s jumping with his hands up is 1.2 meters

Given that there are 10 meters between me and the basket and there are 3 meters between me and my opponent, I must find the quadratic equation where the maximum point will be (3,1.2) or more and passes through (10,0.7). The first coordinate pair is (3,1.2) because x is the distance between me and my opponent and 1.2 is the height of my opponent from the top of my head, which is the origin, and the tip of his/her fingers.

Using these two coordinate pairs I can come up with a quadratic equation. The following is a diagram that illustrates scenario A:
A quadratic equation is the function of a parabola. A parabola is bell shaped, or shaped like a curve, either facing upwards or downwards. In this case our parabola is facing downwards since the ball is being thrown overhand. In order to find the equation of the parabola we must use the two coordinate pairs. A quadratic equation is expressed as \( y = ax^2 + bx + c \). In this case my head is the origin \((0,0)\) and both the basket and my opponent lie in front of me to the right, which means that the parabola will cross the \(y\)-axis at 0. Since \(c\) represents the \(y\)-axis value, then \(c = 0\). This means our equation is now \( y = ax^2 + bx \). To find the equation we must plot both coordinate pairs individually in the equation as such:

- In the coordinate pair \((3,1.2)\), \(x=3\) and \(y=1.2\)
- $1.2 = a(3)^2 + b(3)$
- $1.2 = 9a + 3b$

- In the coordinate pair $(10,0.7)$, $x=10$ and $y=0.7$
- $0.7 = a(10)^2 + b(10)$
- $0.7 = 100a + 10b$

After plotting the coordinate pairs in the quadratic equation we end up with two equations: $1.2 = 9a + 3b$ and $0.7 = 100a + 10b$. We are faced with the obstacle of having two variables, which leads us to simultaneous equations. By solving the two equations simultaneously we will be able to obtain one variable and then use it to find the other variable as such:

$\rightarrow 1.2 = 9a + 3b \times 10$
$0.7 = 100a + 10b \times 3$

$\rightarrow 12 = 90a + 30b$
$2.1 = 300a + 30b \times -1$

$\rightarrow 12 = 90a + 30b$
$-2.1 = -300a - 30b$

$\rightarrow 9.9 = -210a$
$a = -0.047$

$\rightarrow 1.2 = 9(-0.047) + 3b$
$1.2 = -0.423 + 3b$
$1.623 = 3b$
$b = 0.541$

If $a = -0.047$, $b = 0.541$ and $c = 0$, then the final formula is:
\[ y = -0.047x^2 + 0.541x \]

When drawing the formula on a grid it forms the following curve:
Using the equation obtained, the speed of the ball can be determined by finding the derivative of the quadratic equation, as seen below:

\[ y = -0.047x^2 + 0.541x \]
\[ f(x) = -0.047x^2 + 0.541x \]
\[ f'(x) = (2)(-0.047)x + 0.541 \]
\[ f'(x) = -0.094x + 0.541 \]

The derivate \( f'(x) = -0.094x + 0.541 \) is the equation of a tangent at any point on the curve and it represents the speed of the trajectory. Using the speed we can find out if there is enough time from the ball to be thrown and pass the opponent without having him block it.

Since the ball is being thrown from the origin \((0,0)\), we must plot 0 in place of the \(x\) in the equation of the derivative, as follows:

\[ f'(0) = -0.094(0) + 0.541 \]
\[ f'(0) = 0.541 \]

Therefore, the speed of the ball = 0.451\(\text{ms}^{-1}\)

\[ f'(3) = -0.094(3) + 0.541 \]
\[ f'(3) = 0.1681 \]
Vectors

A vector is a quantity that has both size and direction. Vectors are associated with basketball where the speed at which the ball travels forms a vector. For example if the ball is thrown at 5 mph going east, then the ball forms a vector. Then when the ball is thrown from that point in a southwest direction, traveling at 6 mph another vector is formed. The resultant vector, which connects the first two vectors, can be found through mathematics, more specifically, graphic vector addition. The following is an illustration of the vectors formed by the ball thrown. Vector A represents the ball being thrown at 5mph eastward with an $85^\circ$ angle and Vector B represents the ball being thrown at 6 mph northward with and angle of $30^\circ$. Vector R is the resultant vector for which we will find the value.

\[
\begin{align*}
\text{Vector } A &= 5 \text{ at } 85 \\
\text{Vector } B &= 6 \text{ at } 30
\end{align*}
\]

We must start of by finding the vector components. In order to find vector components in this case, we must form a right angle from each vector and use trigonometry.
By combining Vectors A and B, we can find the angle of Vector R and its magnitude:

- $Ax = 5 \cos 85^\circ = 0.44$
- $Ay = 5 \sin 20^\circ = 1.71$
- $Bx = 6 \cos 85^\circ = 0.52$
- $By = 6 \sin 20^\circ = 2.05$

By combining Vectors A and B, we can find the angle of Vector R and its magnitude:

- $Rx = Ax + Bx$
- $Rx = 0.44 + 0.52$
- $Rx = 0.96$

- $Ry = Ay + By$
- $Ry = 1.71 + 2.05$
- $Ry = 3.76$

- $R = \sqrt{(Rx)^2 + (Ry)^2}$
- $R = \sqrt{(0.96)^2 + (3.76)^2}$
- $R = \sqrt{(0.92) + (14.14)}$
- $R = \sqrt{15.06}$
- $R = 3.88$

- $\theta = \tan^{-1} \left(\frac{Ry}{Rx}\right)$
- $\theta = \tan^{-1} (3.92)$
- $\theta = 75.69^\circ$
Therefore, $\mathbf{R} = 3.88 \text{ at } 75.69^\circ$

Vector $\mathbf{R}$ represents the distance that the ball has traveled from the beginning of point $A$ till the end, which is the end of vectors $\mathbf{B}$ and $\mathbf{R}$. As we can see, with one throw the ball travels 3.88 mph in the air at a 75.69 degree angle, whereas with one pass (two throws) it travels 5 mph at 85 degrees then 6 mph at 30 degrees. Although the ball travels slower with a single throw, a basketball player must weigh his/her options given the obstacles and opponents present in order to ensure the best play.
Conclusion

Math is involved in all that surrounds us. Basketball is only one of the sports that involve math. Math like geometry and algebra are very applicable to basketball as the ball forms curves when being shot and the court is full of measurements when involving the players or not. Through this investigation, I have discovered that math is highly involved in the shooting and blocking aspects of basketball. This is important as we can plan how we play to a certain extent based on what we know regarding the different measurements in the court. Also it is interesting to see how by playing basketball we are creating parabolas and our height and physical strength affect these parabolas. Therefore, math is involved greatly in basketball as we have seen in this exploration, we can create graphs by finding equations derived from different measurements and positions we take when playing.
Bibliography
